Who Benefits from Targeted Advertising?

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PRELIMINARY DRAFT PLEASE DO NOT CIRCULATE"

Abstract

We investigate and compare the welfare and allocative effects of alternative consumer data-handling regimes in online targeted advertising. We develop a three-players model that includes firms, consumers, and an intermediary. We analyze the model under various scenarios that differ in the type and amount of consumer data available to the intermediary and to firms. Our results suggest that there are situations in which the intermediary's incentives regarding the type of consumer information to be used for targeting are misaligned with the incentives of firms and consumers. Furthermore, we find that consumer surplus depends on the amount and type information has been exchanged during the targeting process; in particular, consumers surplus may be higher when less and selected type of personal information is made available during the targeting process. Our findings contribute to the ongoing debate over the economic and social implications of the evolution of online tracking and advertising systems.

1 Introduction

In both research and policy circles, a spirited debate has emerged over developments in Internet technologies that allow the collection and analysis of large amounts of consumer information. The aggregation of diverse databases of individuals data, and the application of increasingly sophisticated inferential techniques to those databases, can make services and transactions more efficient, and may help addressing complex societal problems (McAfee and Brynjolfsson, 2012). This so-called "data economy" (and the related notion of "big data") may therefore become a source of innovation, growth, and welfare increases for firms and consumers alike. On the other hand, more data available to firms and decision makers may not always translate to more social progress, economic efficiency, or equality (Crawford et al., 2014). Economic imbalances between consumers and organizations may even increase due to the exacerbation of information asymmetries between data "subjects" and data "holders" (The White House Report, 2015). We contribute to this debate by investigating the extent to which the data economy can *increase* overall economic welfare, and the extent to which it can merely affect the *allocation* of welfare between different stakeholders. We focus on the case of online targeted advertising — one of the most common applications of the market for personal data (Tucker, 2012).

Much has been recently written, in both the academic and the trade literatures, regarding the potential benefits of increasingly widespread and precise collection and usage of consumer data for the targeting of online ads. Data analytics offer marketers and advertising firms the ability to create detailed profiles of consumers, predict their preferences, and serve them the right products or services. Industry representatives have emphasized the benefits that both advertisers and consumers can derive from these technologies: "targeting is not only good for consumers [...] it's a rare win for everyone. [...] It ensures that ad placements display content that you might be interested in rather than ads that are irrelevant and uninteresting. [...] Advertisers [...] achieve higher brand awareness and a greater chance of selling the product. Publishers also win as being able to offer behavioral targeting increases the value of the ad placements and therefore their revenues" (Unanimis Consulting Limited, 2011). In an interview released by AdExchanger (2011),[1] Chad Little, CEO of Fetchback, a company part of Ebay Enterprise, said: "[...] behavioral tracking shouldnt be feared, but instead, embraced. Tracking can simplify and improve a consumer's online experience. By utilizing gathered behavioral data in a strategic manner, online retailers can put the power of the online tracking to work for their consumers." In turn, consumers may not like tracking, but seem to appreciate the importance of an ad-supported Internet. A poll commissioned by the Digital Advertising Alliance in 2013 (Zogby Analystics, 2013) shows that nearly 70 percent of U.S. respondents like at least some ads tailored directly to their interests, compared to only 16 percent who prefer to only see generic ads for products and services.

In short, one of the claim put forward by industry representatives — and, apparently, increasingly accepted by U.S. consumers — is that online targeted ads represent an economic win-win: they reduce search costs for consumers and advertising costs for firms. In this manuscript, we use economic modeling to test those claims. We investigate how the sharing of different types of consumer data can differentially affect the welfare of firms (advertisers), consumers (online users), and an intermediary (the ad exchange) that facilitates the matching between firms and consumers.

Our model focuses on Real-Time bidding, a technology recently introduced that allows the allocation of online display advertisement spaces at real-time through online platforms called Ad Exchanges. Specifically, we focus on the interaction among three players: firms (the advertisers, who compete with each other for consumers' attention), consumers (the online users, who visit websites, are shown targeted ads, and purchase products online), and a monopoly intermediary (the ad exchange). We assume that consumers can be characterized by two categories of information. One category is horizontal information. Horizontal information captures consumers' tastes and products preferences. This category may include information such as gender, past browsing history, interests, and so on. The second cate-

[1] If A Consumer Asked You, "Why Is Tracking Good?", What Would You Say? October 2011, *http : //adexchanger.com/online − advertising/why − is − tracking − good/#valueclick.*

gory is vertical information. It captures differences in consumers' purchasing power. This category may include information such as job occupation, financial information, and even geographical location (as a correlation usually exists between a zip-code an individual lives in and her socio-economic status). Advertisers are firms that produce products and want to advertise them to consumers. Firms buy advertisements by participating in real-time auctions run by the intermediary ad exchange. When a consumer arrives to a website, a signal containing information about that user is sent to the ad exchange; the ad exchange, in turns, sends the signal to the advertiser firms along with consumers' information. Advertisers, on the basis of the information they receive about the user, decide how much to bid. The winner of the auction shows the advertisement to the consumer and pays the second-highest bid.

We consider four scenarios that differ in the amount and/or type of information that is available to the ad exchange and to the advertising firms during the bidding process. First, we consider a benchmark case in which no information about consumers is available. This case corresponds to an extreme full-privacy case where ads are not targeted. Second, we consider the case in which only horizontal information - consumers' brand preferences - is available. Third, we consider the case in which only vertical information - consumers' differences in purchasing power, which allow firms to distinguish between high valuation and low valuation consumers - is available. Finally, we consider the case in which both horizontal and vertical information about consumers can be observed. For each scenario, we use Nash equilibrium strategies to derive each advertisers' bidding strategy and pricing strategy; we then determine the winner of the auction and the final outcome of the game in terms of advertisers' profit, ad exchange's revenues, and consumers' welfare. We perform model simulations to analyze how the outcome of the process changes for the three main players under the different informational scenarios.

Our results can be summarized as follows. First, under general conditions, we find that consumers welfare is higher when only specific type of information is exchanged and, generally, when less information is exchanged. Second, there exist situations in which the incentives of the intermediary are misaligned with respect to consumers' interest; stated differently, the intermediary that acts as profit-maximizing agent may decide to adopt strategies that increase its expected payoff. These findings contribute to the ongoing debate over the economic and social implications of the evolution of online tracking and advertising systems. They have policy implications, because they highlight how the commerce in consumer data can differentially affect the welfare of data holders and data subjects. In particular, they contribute to the ongoing industry and regulatory debate over the economic and social implications of the adoption of increasingly sophisticated tracking and advertising technologies.

2 Related Work

The literature on advertising and, more recently, online targeted advertising, is wide and diverse. First, it can be related to early work on economics of advertising that looked at the impact of advertising on product information and pricing. Butters (1977) proposes a model of informational advertisement, where the role of advertising is to provide information about the existence of the products. Grossman and Shapiro (1984) and Soberman (2004) extend the model to horizontally differentiated markets and analyze the impact of informative advertising on competition and on the availability of different varieties of the same product. Iyer et al. (2005) examines advertising strategy when companies can target advertising to different segments of consumers. They argue that the use of targeted advertising increases the market price and leads to higher profits in comparison to mass or random advertising. Similarly, Esteban and Hernandez (2007) argue that targeting may be seen as an implicit collusion device between firms producing differentiated goods since each firm will advertise only towards its consumers. This implies that, in equilibrium, the entire market will be divided into mutually exclusive captive segments where each firm acts as a pure local monopolist.

Recently, authors in the information system field have offered a more in-depth analysis of the targeted advertising ecosystem by taking into consideration the fact that the targeting process is an intermediated process (Zhang and Catona, 2012), and by considering the important role played by publishers in the targeting process (Chen et al., 2014). Zhang and Catona (2012) analyze how the existence of an independent, profit-maximizer intermediary that sells advertising space and implements the target technology impacts market's outcomes and targeting accuracy. They suggest that, when product market competition is low, the intermediary offers accurate targeting; when product market competition is high, the intermediary offers inaccurate target technology that decreases the ability of the advertisers to create informational differentiation. This leads to an increase in the price competition and gives firms the incentive to bid for the competitors' content topics.

Differently from those works, the focus of our model is on the welfare and allocative aspects of online targeted advertising. The competition among companies is mostly modeled as competition for advertisements' space and consumers allocation. Furthermore, we introduce the possibility that consumers differ along two dimensions: an horizontal one, that captures consumers' products preference; and a vertical one, that captures differences in consumers reservation prices, as some individuals may have higher purchase power than others. Typically, models focus on only one dimension at the time. In this paper, we bring those dimensions together to provide a more complete description of consumers' behavior.

Secondly, our work is related to the IS literature on online auctions and search-advertising (Bapna et al., 2003; Pinker et al., 2003; Katona and Sarvary, 2010; Liu et al., 2010; Chen and He, 2011). Those works study the features that characterizes online auctions. Some of them have specifically focused on the equilibrium properties of the generalized second-price auction, commonly used to place search-advertisements. In this paper, we rely on existing results from auction theory (Vicrey, 1961; Ausubel and Milgrom, 2006) and focus on the Real-Time bidding technology that, introduced in 2007, has been growing really fast and it is expected to take over the market for the allocation of display advertising.

Finally, our work can be related to the literature on information congestion and consumers' privacy. Anderson-De Palma (2009) and Van Zandt (2004) develop models in which firms may send too many messages to consumers, which lowers social welfare because higher-value messages are crowded out by lower value messages. Hann et al. (2008) propose a model in which consumers try to avoid advertising to protect their privacy in two ways: through concealment or deflection. The authors show that while concealment efforts by low-type consumers may reduce total welfare, as more solicitations are shifted toward high-type consumers, deflection efforts reduce the direct privacy harm. Finally, Casadesus-Masanell and Hervas-Drane (2015) develop a model where firms compete for consumer information and derive revenues both from selling products as well as from disclosing consumer information. They find that competition in the market generally decreases the amount of consumer information that is disclosed; nevertheless, higher intensity of competition between firms can increase the stock of information disclosed and reduce consumer privacy.

Similarly to these works, we focus on the effects of targeted advertising on consumers' welfare. Differently, in the first version of the model, we assume that consumers do not use any blocking mechanism but are always exposed to one advertisement.

3 The Model

3.1 Real Time Bidding

Real-time bidding (RTB) is a paradigm of serving ads that aims at bringing more liquidity to the online advertising market. Specifically, RTB allows advertisers to buy online display advertisement spaces at real-time through ad exchanges. The mechanism works as follows. When a user visits a publisher's web site belonging to an ad exchange's network, a request is sent to the ad exchange which subsequently broadcasts it along with user data (user's IP address, geo-location, user's cookies, information about browsing behaviors and others) to ad buyers and holds an auction. Bidders analyze the impression and submit their bid responses. As the ad exchange encourages truthful bidding through the use of second-price auctions, the best strategy for firms is to bid in accordance with their true valuations for the consumer. The winning party is allowed to serve the advertisement to the user and pays the second-highest bid.

The model developed in this paper focuses on three players: i) Advertiser firms. We assume that firms that wish to target specific consumers buy advertisements by participating in online auctions. (For the rest of the paper, we will use the terms advertisers, companies, and firms interchangeably). ii) Ad exchange. We take into consideration the existence of one platform through which RTB and targeting can be implemented. (For the rest of the paper, we will refer to this player as the Intermediary). iii) Consumers. They are the visitors to the website where the ad is displayed and they are the focus of the auction. In other words, advertisers bid for a given consumer, represented by the collection of information that are exchanged and/or made available during the bidding process.

3.2 Basic Setting

Our model considers two firms, i = 1,2 that produce two different products at a constant marginal cost, assumed to be zero without loss of generality. The market consists of a unit mass of consumers. Each consumer has a demand for at most one unit of the product. Consumers differ along two dimensions - horizontal and vertical.

First, each consumer can take one of two *horizontal* positions, capturing the consumer's natural preference for either one of the products. This means that each firm has a segment of consumers who have a preference for its product (that is, everything else equal, their willingness to pay is higher for that product). We denote by v a consumer's reservation price for his favorite product. Stated differently, v represents the maximum amount of money that the consumer is willing to pay for the product he likes the most. Similarly, we denote by w a consumer's reservation price for the other product. In other words, w represents the maximum amount of money that the consumer is willing to pay for a product that is not his favorite. We assume that $v \geq w$. Put simply, the amount of money that a consumer is willing to pay for his favorite product is greater than the amount he is willing to pay for a different product. We assume that a proportion α_1 of consumers prefers Firm 1 and a proportion α_2 prefers Firm 2, with $\alpha_1 + \alpha_2 = 1$ and $0 \leq \alpha_i \leq 1$ with $i = 1, 2$. Those proportions are known to the firms. Differently, a firm does not know whether a specific consumer likes its product better than the other without any extra information. In other words, the only piece of information that a firm has is the probability of observing a consumer with a preference for its product; probability that is captured by the proportion α_i .

Second, each consumer can take two *vertical* positions, capturing their differences in purchasing power. A consumer can be a low valuation consumer or a high valuation consumer. To better understand what we mean by vertical position, let us consider an example. Consider two consumers, both with a preference for the product of Firm 1. According to our assumptions, this implies that both consumers are willing to spend more for product 1 than for product 2. Nevertheless, even though the two consumers have the same product preference, they may have different purchasing powers, in the sense that one can afford to pay more than the other for the product he likes. For instance, the consumer with higher purchasing power (which we call "high valuation" consumer) can afford to pay a maximum of \$ 10. Differently, the consumer with lower purchasing power (that we call "low valuation" consumer) can only afford to pay a maximum of \$ 8.[2] This has consequences on the amount that the consumers are willing to pay for product 2. By definition, both consumers will be willing to spend less on product 2 because that is not their favorite product. Nevertheless, following the example, the high valuation consumer can pay a maximum of \$ 7 for product 2 while the low valuation consumer can pay a maximum of \$ 5 for product 2.

From a notational point of view, we denote by v_h the amount that a high valuation consumer is willing to pay for his favorite product and we denote by w_h the amount he is willing to pay for the other product. Similarly, we denote by v_l the amount that a low

[2]The difference between showing a preference for a product and the actual amount a consumer can afford to pay is crucial in online settings. For instance, a user with a passion for cars may often browse websites about sport's cars; therefore, one can infer that the consumer has a preference for sport's cars. Nevertheless, that does not mean that the consumer can actually afford to pay for a sport's car. As a consequence, the separation between horizontal and vertical information is fundamental.

valuation consumer is willing to pay for his favorite product and we denote by w_l the amount he is willing to pay for the other product. Note that, while we assume that $v_h \geq v_l$ and $w_h \geq w_l$, we do not make any assumption on the relationship between v_l and w_h. This makes our model flexible, and allows us to analyze what happens when different market configurations are considered. The only assumption we make is that $v_l/v_h = w_l/w_h$. We assume that a proportion β of consumers is a high valuation consumer and a proportion $1-\beta$ is a low valuation consumer, with $0 \leq \beta \leq 1$. As before, firms have information about β. Differently, without any extra information, a firm does not know whether a specific consumer is high valuation or low valuation. In other words, the only piece of information that a firm has, a priori, is the probability of observing a high valuation consumer — probability that is captured by the proportion β.

In this model, consumers have preferences over products. Without advertising, however, they do not know which company sells which product and at what price. In this sense, advertising plays an informative role, as it informs consumers of the existing firms and prices. Given the market's structure, a firm's objective is to target consumers that have a preference and, therefore, a high reservation price for its product . Nevertheless, companies cannot target consumers directly as they cannot "identify" consumers without extra information (the only pieces of information they have are the proportions α and β). We can think of companies obtaining those information through a simple analysis of their specific market. As a consequence, they have to rely on an intermediary (the online ad exchange platform) that possess tracking and targeting technologies and runs a second-price auction to allocate consumers between companies. Importantly, during the auction, the ad exchange may make available to the firms additional pieces of information about the consumers. Firms, after observing the set of information about the consumer, decide how much to bid for the advertisement and the pricing strategy for their product. The highest bidder wins and pay the bid of the second company. The fact that firms set their price along with the bidding strategy may seem a strong assumption; in fact, that assumption is equivalent to saying that firms can offer personalized discounts to consumers, which is realistic.

In the following sections, we consider and then compare four cases that differ in the amount and type of information that companies see about the consumer before submitting the bid: i) No Information. Firms are not able to observe any information about consumers. ii) Only Horizontal Information. Firms are able to observe which product a consumer prefers but they do not observe the vertical position (purchasing power). iii) Only Vertical Information. Firms are able to observe whether the consumer is a high valuation or low valuation consumer but they do not observe his horizontal position (product's preference). iv) Both Horizontal and Vertical information. Firms obtain all the information about the consumer — that is, his product's preference and purchasing power. In the analysis that follows, we consider those four scenarios and we derive each firm's bidding strategy and pricing strategy under each scenario. We then analyze how the outcome of the game changes in terms of firms' profits, ad exchange's profit, and consumers' welfare.

Before proceeding with the analysis, let us clarify those variables. Since we assume a marginal cost of production equal to zero, a firm's profits are simply given by revenues (if the consumer buys the product) minus the amount paid by the company if she wins the online auction for the advertisement (amount equal to the second-highest bid). As for the ad exchange, we do not consider any cost of running the auction. Hence, ad exchange's profits are simply equal to the amount paid by the firm that wins the online auction. Finally, we conservatively define consumer welfare as the difference between the maximum amount a consumer is willing to pay for a product and the price he actually pays for it.

3.3 Sequence of Events

The sequence of events in the model is as follows:

1) At any given point in time, a consumer arrives to a website. The consumer is characterized by two pieces of information: horizontal and vertical.

2) The ad exchange receives a signal that the specific consumer is online and turns the signal

to advertisers that wish to show advertisements to that consumer.

3) The ad exchange runs a second-price auction to allocate the advertisement space. During the auction, it may make visible to the advertisers all or part of the information about the consumer (only the horizontal information, only the vertical information, or both).

4) On the basis of the information observed, advertisers decide how much to bid and set the price of the product.

5) The firm that submits the highest bid wins the auction, pays the second-highest bid and shows the ad to the consumer.

6) The consumer sees the ad and decide whether or not to buy the product. The consumer buys as long as the price is lower than his reservation price.

Figure 1 offers a visual representation of the sequence of the events.

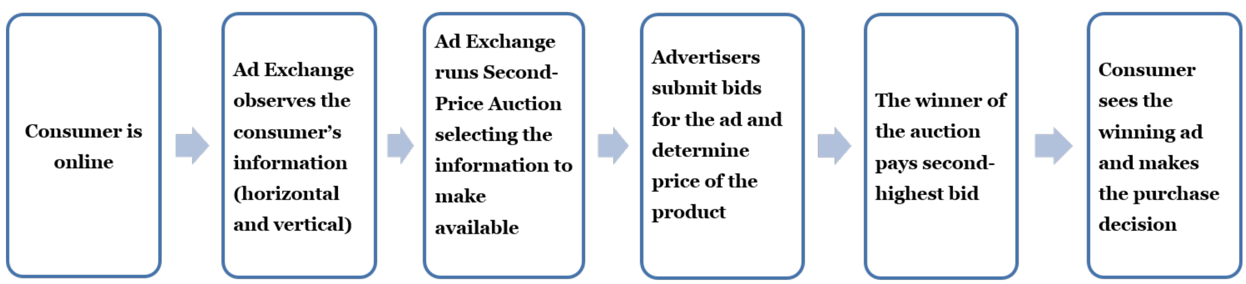

Figure 1: Sequence of Events

4 Analysis

In this section, we develop the analysis for the four informational cases described above. We start considering the benchmark case where no information about individual consumers is available. In this scenario, companies have only the common information on the market's structure; therefore, their bidding and pricing strategies will be based on their expectation about the consumer's willingness to pay. Given the flexibility allowed by the model, we have two different cases: 1) $v_l \geq w_h$; 2)$v_l < w_h$.

When $v_l \geq w_h$, the firm can choose among the following possible strategies: i) a firm can try to capture the whole market by bidding $b_i = w_l$ and setting $P_i = w_l$; ii) a firm can decide to capture only his segment of consumers and, among those, the high valuation ones, by bidding $b_i = \alpha_i \beta v_h$ and setting $P_i = v_h$; ii) a firm can decide to capture his all segment of consumers (high and low valuation) by bidding $b_i = \alpha_i v_l$ and setting $P_i = v_l$; iv) finally, if β is large enough, the company may try to capture only the high valuation consumers by bidding $b_i = \beta w_h + (1-\beta)\alpha_i w_h$ and setting $P_i = w_h$.

When $v_l < w_h$, the first two strategies are equivalent to the ones just described. The last two are different as: if the company decides to capture all his segment of consumers (high and low valuation) by setting $P_i = v_l$, it will now capture also part of the high valuation of the other segment (since $v_l < w_h$). Therefore, the bidding strategy will be $b_i = \beta v_l + \alpha_i(1-\beta)v_l$. Finally, if β is large enough and the company tries to capture all the high valuation consumers by $P_i = w_h$, it will not be able to capture also the proportion of his low valuation consumers, as $v_l < w_h$. Therefore, the bidding strategy will be $b_i = \beta w_h$. Among the strategies we described, firms choose the one that yields the highest expected revenue. This depends on the value of the parameters α and β that determine the market structure and on the consumers' valuations. When the market is symmetric, that is $\alpha_i = \alpha_j$, both firms will bid the same and profit will be zero in expectation.

LEMMA 1. *When firms do not observe extra information about consumers:*

- *if* $v_l \geq w_h$*: firm i bid is* $b_i = max(w_l, \alpha_i v_l, \alpha_i \beta v_h, \beta w_h + (1-\beta)\alpha_i w_h)$ *and firm j bid is* $b_j = max(w_l, \alpha_j v_l, \alpha_j \beta v_h, \beta w_h + (1-\beta)\alpha_j w_h)$.

- *if* $v_l < w_h$*: firm i bid is* $b_i = max(w_l, \beta v_l + (1-\beta)\alpha_i v_l, \alpha_i \beta v_h, \beta w_h$ *and firm j bid is* $b_j = max(w_l, \beta v_l + (1-\beta)\alpha_j v_l, \alpha_j \beta v_h, \beta w_h$.

- *When the market is symmetric.* $b_i = bj$ *and, in expectation, firm's profit will be zero.*

- *The intermediary revenue is* $min(b_i, bj)$*.*

The no information scenario can be related to an untargeted, random advertising technology: firms cannot target specific consumers. As a consequence, firms act in expectation but, by so doing, they may tend to bid more than what they should for a given advertisement and consumer. Since both firms have an incentive to bid, the intermediary's profit is positive and equal to the second higher bid.

Next, we consider the case in which only the consumer' horizontal information is available to companies during the auction. Let us assume that companies observe a consumer of type 1. Firm 1 can decide to adopt one of two strategies: it can choose to capture any consumer coming from its segment, by setting $p_1 = v_l$. In this case, the bid would be $b_1 = v_l$. Alternatively, if β is large enough, it can choose to capture, inside its segment, only the high valuation consumers by setting $p_1 = v_h$. In this case, the bidding strategy would be $b_1 = \beta v_h$. In summary, Firm 1 bidding strategy is $b_1 = max(\beta v_h, v_l)$.
Note that also Firm 2 submits a positive bid. Indeed, since the consumer does have a positive valuation for its product too, it will follow a bidding strategy similar to company 1, by bidding $b_2 = max(\beta w_h, w_l)$. Those results are summarized in Lemma 2. The proof is contained in Appendix 1.

LEMMA 2. *When companies observe a consumer of type* i*:*

- *firm* i *bids* $b_i = max(\beta v_h, v_l)$ *and sets a price equal to* $p_i^* = v_h$ *if* $\beta v_h \geq v_l$ *and* $p_i^* = v_l$ *otherwise. Firm* j *(with* $i, j = 1, 2$ *and* $i \neq j$*) bids* $b_j = max(\beta w_h, w_l)$ *and sets a price equal to* $p_j^* = w_h$ *if* $\beta w_h \geq v_l$ *and* $p_j^* = w_l$ *otherwise.*

- *Company* i *always wins the auction for consumer of type* i*, as* $b_i = max(\beta v_h, v_l)$ *is always greater than* $b_j = max(\beta w_h, w_l)$*.*

- *The Intermediary revenue is $max(\beta w_h, w_l)$.*

When only the horizontal information is available, even though both companies submit a positive bid, company i that observes a consumer of type i always wins the auction. Consequently, consumers are always shown the advertisement for their favorite product. Since in this case also the other company submits a positive bid, the intermediary revenue is not zero.

The next scenario we consider is the case in which only the vertical information is available to companies during the auction: firms can distinguish between high valuation and low valuation consumers, but they do not know consumer's preferences. Let us assume that the consumer is high valuation. Firms know that the consumer will be willing to pay v_h for his favorite product and w_h for the other. They also know the respective probability of getting a consumer from a specific segment, that is α. Hence, they can decide to adopt one of two strategies: i) company i can decide to capture any consumer that is high valuation by setting a price equal to w_h; ii) or, it can decide to capture any consumer coming from its segment and that is high valuation, by setting a price equal to v_h. The same reasoning can be applied to low valuation consumers. Importantly, in this case, it is not immediately clear which company is going to win the auction: the final outcome depends on the parameters' values. Let us consider, for instance, a high valuation consumer. If the market is asymmetric ($\alpha > 0.5$) and the difference between v_h and w_h is large enough, company i with the largest segment of consumers bids $\alpha_i v_h$ and wins the auction, with $i = 1, 2$. There are two main cases in which both companies submit the same bid and the consumer is randomly assigned: i) the market is symmetric and both submits αv_h; ii) the difference between v_h and w_h is not large enough and both submit w_h. The same reasoning is valid for a low valuation consumer. Lemma 3 summarizes the result. The proof is provided in Appendix 1.

LEMMA 3. *When only consumers' vertical position is available:*

- *Both companies submit a positive bid.*

- *If consumer is high valuation, company i sets $p_i^* = v_h$ and bid $b_i^* = \alpha_i v_h$ if $\alpha_i v_h \geq w_h$; the company sets $p_i^* = w_h$ and bid $b_i^* = w_h$ otherwise.*

- *If consumer is low valuation, company i sets $p_i^* = v_l$ and bid $b_i^* = \alpha_i v_l$ if $\alpha_i v_l \geq w_l$; the company sets $p_i^* = w_l$ and bid $b_i^* = w_l$ otherwise.*

- *If the market is asymmetric and v_h is sufficiently bigger than w_h, firm i with the larger consumers' segment wins the auction and gets a consumer of type i with probability α_i. The Intermediary revenue is $max\{\alpha_j v_h, w_h\}$ if the consumer is high valuation and $max\{\alpha_j v_l, w_l\}$ if the consumer is low valuation.*

- *If the market is symmetric, both companies submit the same bid and the consumer is randomly assigned. The Intermediary revenue is $max\{\frac{1}{2}v_h, w_h\}$ or $max\{\frac{1}{2}v_l, w_l\}$ depending on whether the consumer is high or low valuation.*

. Finally, under the last case we consider, both horizontal and vertical information about the consumer is available to companies during the auction. Let as assume firm observe a high valuation consumer of type 1. Then we have $b_1 = v_h$ and $p_1 = v_h$, for firm 1. For firm 2, we have $b_2 = w_h$ and $p_2 = w_h$. Similarly, if the consumer is low valuation we have that that $b_1 = v_l$ and $p_1 = v_l$, for firm 1; for firm 2 we have $b_2 = w_l$ and $p_2 = w_l$.

LEMMA 4. *When companies can observe both horizontal and vertical information:*

- *Firm i always wins the auction for consumer i.*

- *The Intermediary revenue is w_h if the consumer is high valuation and w_l if the consumer is low valuation.*

In this last scenario, companies are able to observe both a consumer's product preference and his purchasing power. Consequently, if firm 1 observes, for instance, a high valuation consumer of type 1, it can set the product's price to be exactly equal the consumer's reservation price, vh, and it also submits a bid equal to vh (the firms expects that the consumer will buy its product at that price). Similarly, firm 2 knows that the consumer is only willing to pay wh for its product; hence, it sets the price of product and the bid for the advertisement accordingly. Since by assumption we have that vh wh, firm 1 that bids vh always wins the auction for the consumer. The same reasoning applies to a consumer that is low valuation.

5 Optimal Information Policy

In this section, we analyze how the players' benefits - defined as firms' profits, intermediary's profit and consumers' welfare - change across the different informational scenarios. In other words, we examine how differential access to consumers data may differentially affect the welfare of various stakeholders. To exemplify our reasoning, let us consider the case of a consumer which is high valuation of type i and let us examine how his surplus changes when different pieces of information about him are made available. We will also consider a symmetric market, that is a market where $\alpha_i = \alpha_j$.

In the horizontal information case, we found before that company i is always going to win the auction for consumer i. In other words, the consumer sees the advertisement for his favorite product. If company i sets a price equal to v_h, the consumer surplus is 0; but if company i sets a price equal to v_l, the consumer surplus is positive and equal to $v_h - v_l$.

In the vertical information case, we derived that when the market is symmetric, companies submit the same bid. Depending on the market structure, price is either going to be v_h, leaving consumers with no surplus, or w_h, leaving consumers with a positive surplus.

In the No information case, price varies depending on firm's expectation. Unless the price is v_h, the high valuation consumer is going to end up with a positive surplus.

Finally, in the complete information case, consumer of type i is always going to see the advertisement for its favorite product and it always ends up with zero surplus.

From this example, it should become clearer that which informational scenario is better for the consumer — that is, the scenario that yields the highest expected surplus — depends on the market structure and consumers' composition. The following proposition summarizes the results.

PROPOSITION 1. For the consumers, the optimal information policy is as follows:

- when $\beta v_h \leq v_l$, $\alpha v_h \geq w_h$ and $w_h/v_l \leq \frac{\alpha}{\alpha+\beta-\alpha\beta}$, revealing the Horizontal Information is optimal for the consumers.
- When $\beta v_h \geq v_l$, $\alpha v_h \leq w_h$ and $w_h/v_l > \frac{\alpha+\beta-\alpha\beta}{\beta}$, revealing the Vertical Information is optimal for the consumers.
- When $\beta v_h \leq v_l$, $\alpha v_h \leq w_h$, $\beta \geq (1-\alpha)$ and $w_l/w_h \geq \alpha+\beta-\alpha\beta$ or $w_l/v_l \geq \alpha+\beta-\alpha\beta$, revealing no additional information is optimal for the consumers.
- When $\beta v_h \geq v_l$, $\alpha v_h \geq w_h$ and $w_h/v_h \leq \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$ or $v_l/w_h \leq \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$, consumers are indifferent.
- Otherwise, there is no clear dominance.

Figure 2 offers a visual representation of the different regions described by Proposition 1. On the horizontal axis, the ration w_h/v_h represents the degree of horizontal differentiation, that is how different consumers are in their tastes and preferences; on the vertical axis, the ratio v_l/v_h captures the degree of vertical differentiation, that is how different consumers are in their purchase power. Important to note, when the ratios are high the degree of differentiation is low and viceversa. For instance, Region 2, in red, represents the combination of parameters for which revealing the vertical information is optimal for the consumers. Intuitively, that region is characterized by high degree of vertical differentiation and low degree of horizontal differentiation.

For the other players (the intermediary and the firms), the situation is more straightforward. The firm's expected revenue is always greater in the complete information case. Intuitively, a company is better off when it can perfectly "identify" his consumer and set

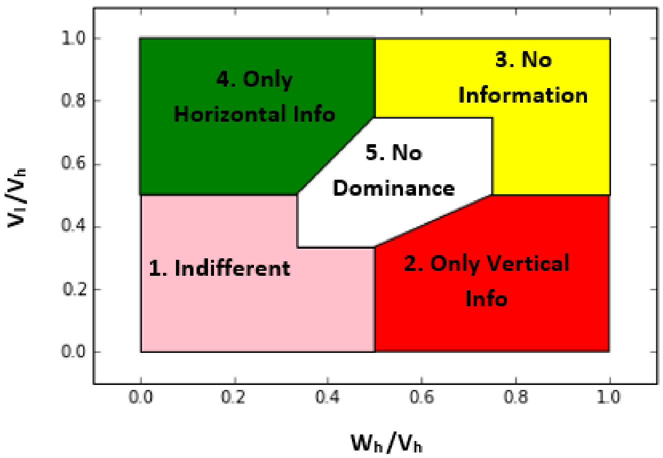

Figure 2: Optimal Information Policy for Consumers

a personalized price. The intermediary expected revenues are always greater in the vertical information case. Indeed, when the Vertical Information is revealed, the competition between companies increases and, in expectation, they tend to bid more aggressively than what they would if the Horizontal Information was revealed. The result for the Intermediary is particularly important; indeed, if the intermediary has power over the consumers' information then, in equilibrium, it will always reveal the information that maximizes its expected revenues. Nevertheless, this may not coincide with the informational regime that maximizes consumers' welfare as well. In other words, the incentive of the two players may be misaligned. Proposition 2 and Corollary 1 formalize these results.

PROPOSITION 2. *Intermediary Choice. In Equilibrium, an Intermediary that has power over the information will always have the incentive to reveal the Vertical Information.*

COROLLARY 1. Let us define the following consumer's surplus loss function:

$$SL_i(r) = W_i^*(r) - W_i(v)$$

where $W_i^*(r)$ is the maximum surplus the consumer's can obtain in region r, for $r = 1, 2, 3, 4$; and $W_i(r)$ is the surplus the consumer's obtain in region r, given the Intermediary's equilibirum choice. Stated differently, the function captures the loss in consumers' surplus due to the fact that in equilibrium the Intermediary may choose to reveal information different from the one that would maximize consumers' surplus. We have the following situations:

- $SL_i(1) = W_i^*(1) - W_i(1) = 0$. *Consumers are Indifferent.*

  Consumers do not suffer any surplus loss because they are indifferent with respect to which informational regime is chosen by the Intermediary.

- $SL_i(2) = W_i^*(2) - W_i(2) = 0$. *The information regime chosen by the Intermediary maximizes consumers' surplus.*

  Consumers do not suffer any surplus loss because the informational regime chosen by the Intermediary coincides with the regime that maximizes consumers' surplus.

- $SL_i(3) = W_i^*(3) - W_i(3) = w_h - w_l$. *The intermediary reveals too much information.*

  Consumers suffer a positive surplus loss because the informational regime chosen by the Intermediary diverges from the regime that maximizes consumers' surplus. Specifically, in that region, consumers would be better off if no information was revealed because more consumers would enjoy a lower price.

- $SL_i(4) = W_i^*(4) - W_i(4) = (1 - \alpha)\beta(v_h - v_l)$. *The intermediary reveals the wrong type of information.*

  Consumers suffer a positive surplus loss because the informational regime chosen by the Intermediary diverges from the regime that maximizes consumers' surplus. Specifically, in that region consumers would be better off if the Horizontal Information was revealed. Indeed, revealing the Horizontal Information would ensure a better product matching and a lower price.

6 Limitations and Possible Extensions

The model we propose can be extended in various ways. First of all, some of the assumptions on consumers behavior may look stylized and can be relaxed. We assume that consumers see only one advertisement at the time, either for firm 1 or for firm 2. We can extend the model by considering the existence of n competing firms and by allowing the possibility that a consumer is shown more than one advertisement at the time. Furthermore, in the proposed version of the model, a consumer buys as long as the price of the advertised product is no greater than the consumer's reservation price for that product. We could relax this assumption by including consumers search behavior to account for the fact that consumers may decide to shop around before buying.

In addition, consumers are not able to block or avoid advertisements. Introducing consumers' ability to control the type and amount of information that is being collected online seems to be an interesting and relevant case, particularly considering that online users can currently rely on various privacy-enhancing technologies. Concerning the assumptions about the ad exchange, we currently consider the existence of a monopoly intermediary. We can allow some degree of competition also on the intermediary side and consider the possibility that companies decide to enter different ad networks.

Finally, the proposed model takes into consideration the interaction between three types of players: advertisers, intermediary and consumers. The online advertising ecosystem is more complex and includes additional subjects. For instance, companies that want to participate to auctions for online advertising usually rely on Demand Side Platform (DSP) that serves advertisers or ad agencies by bidding for their campaigns in multiple ad networks automatically. On the other side, Supply Side Platforms (SSP) serves publishers by registering their inventories (ads space) in different ad networks and accepting the most beneficial automatically. Taking into consideration the existence of those players may help offering a more complete analysis of the online advertising market.

7 Conclusion and Policy Implications

In this paper, we analyzed the welfare impact of targeted advertising by developing an analytical model based on Real-Time Bidding. The model focuses on the interaction among three types of players: firms (the advertisers, who compete with each other for consumers' attention), consumers (the online users, who visit websites, are shown targeted ads, and purchase products online), and an intermediary (the ad exchange platform, which tracks consumers data and serves them the firms' ads). We assumed that consumers are characterized by two dimensions: horizontal information, that captures consumers' preferences for specific products; and vertical information, that captures differences in consumers' purchasing power. Advertisers, firms that produce products and want to advertise them to consumers, buy advertisements by participating in real-time auctions run by the intermediary.

We considered four scenarios that differed in the amount and/or type of information that is available on the market and that advertising firms have available during the bidding process: i) The case in which only the horizontal information, that is which product a consumer prefers, is available. ii) The case in which only the vertical information (whether the consumer is high valuation or low valuation) is available. iii) The case in which both the horizontal and the vertical information about consumers is available. iv) A case in which no information about consumers is available, corresponding to a benchmark case of full privacy protection. For each of the four scenarios, we derived each advertisers' bidding strategy and pricing strategy; we then determined the winner of the auction and the final outcome of the game in terms of advertisers' profit, ad exchange's revenues and consumers' welfare. We found that, under general conditions, consumers welfare is higher when only specific type of information is exchanged and, generally, when less information is exchanged. Furthermore, there exist situations in which the incentives of the intermediary are misaligned with respect to consumers' interest; stated differently, the intermediary that acts as profit-maximizing agent may decide to adopt strategies that increase its expected payoff.

By illustrating how different degrees of consumer data tracking and sharing can differentially affect the welfare of data holders and data subjects, these findings can contribute to the ongoing industry and regulatory debate over the economic and social implications of the adoption of tracking and advertising systems. Notably, these results do not imply that the collection of consumer information should be prohibited - rather, they suggest that there is information which is beneficial for consumers to share, and other information which, instead, could be used by others to the consumer's detriment. Furthermore, what emerges from the model is a scenario alternative to the economic "win-win" often heralded as the likely and desirable outcome of the increased collection and trade of consumers' data: we find, instead, that different players that operate in the ecosystem may have contrasting interests. In turn, such interests may create incentives for practices that are not transparent. For instance, an intermediary that has the power to control which type of information to highlight to other parties (such as advertising firms) during the auction process, may have the incentive to act strategically, by revealing the information that ensure him the highest expected return. According to our analysis, this may be against consumers' best interest.

APPENDIX 1

PROOF OF LEMMA 2. We assume companies observe a consumer of type i, with $i = 1, 2$. We start from the pricing strategy.

Firm i chooses the price to maximize his expected profit, that is:

$$argmax\{(p_i * Prob.Buying) - b_j\}$$

where p_i is the price set by company i, Prob.Buying is the probability that the consumer is going to buy the product and b_j is the bid submitted by company j.

Company i can set two different prices: v_h or v_l. If it sets $p_i = v_l$, its expected revenue is

$$(v_l * 1) - b_j \tag{1}$$

Indeed, in that case the probability that the consumer buys is equal to one, as the consumer sees the advertisement for his favorite product and his reservation price is $\geq v_l$. If it sets $p_i = v_h$, its expected profit is:

$$(v_h * \beta) - b_j \tag{2}$$

In this case, not all the consumers are going to buy the product but only the proportion with high valuation that is β. Putting together equation (1) and (2), we have that company i sets $p_i = v_h$ if $\beta v_h \geq v_l$ and sets $p_i = v_l$ otherwise.

Next, let us consider the bidding strategy. Our results are based on the fact that in second-price auctions, truthful bidding is a dominant strategy. For the result to hold in this case, it is sufficient to introduce an infinitesimal probability that companies do not know who they are competing with. Holding this condition, company i strategy will be to bid its truthful valuation for consumer i, that is equal to the revenue the company expects to gain if that consumer buys the product. When company i sets a price equal to v_l, the expected revenue is also v_l; when it sets a price equal to v_h, its expected revenue is equal to βv_h. Consequently, company i bidding strategy is to bid

$$b_i = max\{\beta v_h, v_l\}$$

When companies can observe consumers' horizontal information, firm i wins the auction for consumer i. We have to consider four different cases.

Let us assume $b_i = \beta v_h$ and $b_j = \beta w_h$. Then $b_i \geq b_j$ as, by assumption, $v_h \geq w_h$. The same conclusion holds if $b_j = w_l$. Indeed, if $b_i = \beta v_h$, it means that $\beta v_h \geq v_l$; since, by assumption, $v_l \geq w_l$, then it must also be that $\beta v_h \geq w_l$. Hence, $b_i \geq b_j$.

Next, let us assume that $b_i = v_l$ and $b_j = \beta w_h$. We know that, by assumption, $v_l \geq w_h$. Since $\beta \leq 1$, we also have that $v_l \geq \beta w_h$. The same result holds if $b_j = w_l$ as again, by assumption, $v_l \geq w_l$.

PROOF OF LEMMA 3.a We assume that the consumer is high valuation consumer. Let us start from the pricing strategy. Firm i chooses the price to maximize his expected profit, that is:

$$argmax\{(p_i * Prob.Buying) - b_j\}$$

where p_i is the price set by company i, Prob.Buying is the probability that the consumer is going to buy the product and b_j is the bid submitted by company j with $i, j = 1, 2$ and $i \neq j$.

Company i can set two different prices: v_h or w_h. If it sets $p_i = v_h$, its expected revenue is

$$(v_h * \alpha_i) - b_j \tag{3}$$

Indeed, in that case the probability that the consumer buys is equal to the probability that the consumer is type i, that is α_i. If it sets $p_i = w_h$, its expected profit is:

$$(w_h * 1) - b_j \tag{4}$$

In this case, all the high valuation consumers are going to buy the product. Putting together equation (3) and (4), we have that company i sets $p_i = v_h$ if $\alpha_i v_h \geq w_h$ and sets $p_i = w_h$ otherwise.

Next, let us consider the bidding strategy. Our results are based on the fact that in second-price auctions, truthful bidding is a dominant strategy. For the result to hold in this case, it is sufficient to introduce an infinitesimal probability that companies do not know who they are competing with. Holding this condition, company i strategy will be to bid its truthful valuation for consumer i, that is equal to the revenue the company expects to gain if that consumer buys the product. When company i sets a price equal to w_h, the expected revenue is also w_h; when it sets a price equal to v_h, its expected revenue is equal to $\alpha_i v_h$. Consequently, company i bidding strategy is to bid $b_i = max\{\alpha_i v_h, w_h\}$.

PROOF OF LEMMA 3.b When companies can observe only the vertical information, firm i that bids $b = \alpha_i v_h$ wins the auction and gets a consumer of type i with probability α_i.

Let us assume that $b_1 = \alpha_1 v_h$, implying that $\alpha_1 v_h \geq w_h$. This happens either when α_1 is large enough or v_h is sufficiently higher than w_h. If $\alpha_1 > \alpha_2$, then firm 1 is always going to win the auction no matter firm's 2 bid. Indeed, if $b_2 = \alpha_2 v_h$, then we have that $b_1 > b_2$ as $\alpha_1 > \alpha_2$. If $b_2 = w_h$, then, again, $b_1 > b_2$ as $\alpha_1 v_h \geq w_h$.

If $\alpha_1 = \alpha_2$ then the two firms submit the same bid. Indeed, in that case, we have that $\alpha_1 v_h = \alpha_2 v_h = \alpha v_h$. Hence, whenever $\alpha v_h \geq w_h$, both firms bid αv_h and viceversa.

PROOF OF LEMMA 4. Follows trivially from the assumption that $v_h \geq w_h$ and $v_l \geq w_l$.

PROOF PROPOSITION 1.

1) *When* $\beta v_h \leq v_l$*,* $\alpha v_h \geq w_h$ *and* $w_h/v_l \leq \frac{\alpha}{\alpha+\beta-\alpha\beta}$*, revealing the Horizontal Information is optimal for the consumers.*

The region is characterized by the following relationships: $\beta v_h \leq v_l$; $\alpha v_h \geq w_h$; and $v_l \geq w_h$. This implies the following outcomes in the different scenarios:

- Horizontal Information: $WinningBid = v_l$, $Price = v_l$, $CSh = \beta(v_h - v_l)$.
- Vertical Information: $WinningBid = \alpha v_h$, $Price = v_h, v_l$, $CSv = \alpha\beta(v_h - v_h) + \alpha(1 - \beta)(v_l - v_l)$.
- No Information: $WinningBid = max\{w_l, \alpha\beta v_h, \alpha v_l, \beta w_h + (1-\beta)\alpha w_h\}$. From previous conditions we know that $\alpha v_h \geq w_h$ implies that $\alpha v_l \geq w_l$, therefore w_l cannot be the maximum. Also, $\beta v_h \leq v_l$ implies that $\alpha\beta v_h$ cannot be the maximum either. Therefore, the max can be either αv_l or $\beta w_h + (1-\beta)\alpha w_h$.

  a) When $w_h/v_l \leq \frac{\alpha}{\alpha+\beta-\alpha\beta}$, the winning bid is αv_l, $Price = v_l$ and $CSn = \alpha\beta(v_h - v_l)$.
  b) When $w_h/v_l > \frac{\alpha}{\alpha+\beta-\alpha\beta}$, the winning bid is $\beta w_h + (1-\beta)\alpha w_h$, $Price = w_h$ and $CSn = \alpha\beta(v_h - w_h) + \alpha(1-\beta)(v_l - w_h) > 0$.

Therefore, when $w_h/v_l \leq \frac{\alpha}{\alpha+\beta-\alpha\beta}$: $CSh > CSn > CSv = CSb = 0$ and $I_h = \{i|V_{ij}(h) - P_{ij}(h) > 0\}$ and $I_n = \{i|V_{ij}(n) - P_{ij}(n) > 0\}$ with $I_n \subset I_h$.

2) *When* $\beta v_h \geq v_l$*,* $\alpha v_h \leq w_h$ *and* $v_l/w_h \leq \frac{\beta}{\alpha+\beta-\alpha\beta}$*, revealing the Vertical Information is optimal for the consumers.*

The region is characterized by the following relationships: $\beta v_h \geq v_l$ and $\alpha v_h \leq w_h$. Also, $v_l < w_h$.This implies the following outcomes in the different scenarios:

- Horizontal Information: $WinningBid = \beta v_h$, $Price = v_h$, $CSh = \beta(v_h - v_h) = 0$.

- Vertical Information: $WinningBid = w_h, w_l$, $Price = w_h, w_l$, $CSv = \alpha\beta(v_h - w_h) + \alpha(1-\beta)(v_l - w_l) > 0$.

- No Information: $WinningBid = max\{w_l, \alpha\beta v_h, \alpha v_l + \beta(1-\alpha)v_l, \beta w_h$. From previous conditions we know that $\beta v_h \geq v_l$ implies that $\beta w_h \geq w_l$, therefore the maximum cannot be w_l. Also, $\alpha v_h \leq w_h$ implies that $\beta w_h \geq \alpha\beta v_h$ and $\alpha\beta v_h$ cannot be the maximum. Therefore, the maximum can be either $\alpha v_l + \beta(1-\alpha)v_l$ or βw_h.
  a) When $w_h/v_l > \frac{\alpha+\beta-\alpha\beta}{\beta}$, the winning bid is βw_h, $Price = w_h$, $CSn = \alpha\beta(v_h - w_h) > 0$.
  b) When $w_h/v_l \leq \frac{\alpha+\beta-\alpha\beta}{\beta}$, the winning bid is $\alpha v_l + \beta(1-\alpha)v_l$, $Price = v_l$ and $CSn = \alpha\beta(v_h - v_l) + (1-\alpha)\beta(w_h - v_l) > 0$.

Therefore, when $w_h/v_l > \frac{\alpha+\beta-\alpha\beta}{\beta}$: $CSv > CSn > CSh = CSb = 0$; $I_n = \{i|V_{ij}(n) - P_{ij}(n) > 0\}$ and $I_v = \{i|V_{ij}(v) - P_{ij}(v) > 0\}$ with $I_n \subset I_v$.

3) *When $\beta v_h \leq v_l$, $\alpha v_h \leq w_h$, $\beta \geq (1-\alpha)$ and $w_l/w_h \geq \alpha + \beta - \alpha\beta$ or $w_l/v_l \geq \alpha + \beta - \alpha\beta$, revealing no additional information is optimal for the consumers.*

The region is characterized by the following relationships: $\beta v_h \leq v_l$ and $\alpha v_h \leq w_h$. We can have $v_l \geq w_h$ or $v_l < w_h$. This implies the following outcomes in the different scenarios:

- Horizontal Information: Winning bid $= v_l$, $Price = v_l$, $CSh = \beta(v_h - v_l) > 0$.

- Vertical Information: Winning Bid $= w_h, w_l$, $Price = w_h, w_l$, $CSv = \alpha\beta(v_h - w_h) + \alpha(1-\beta)(v_l - w_l) > 0$.

- No Information: we have two different maximization problems depending on whether $v_l \geq w_h$ or $v_l < w_h$.
  a) For $v_l \geq w_h$, winning bid $= max\{w_l, \alpha\beta v_h, \alpha v_l, \beta w_h + (1-\beta)\alpha w_h\}$. From the initial conditions, we know that $\alpha v_h \leq w_h$ implies that $w_l \geq \alpha v_l$, therefore αv_l cannot be the

maximum. Also, $\beta v_h \leq v_l$ implies that $\alpha v_l \geq \alpha\beta v_h$, implying again that $w_l \geq \alpha\beta v_h$; as a consequence, $\alpha\beta v_h$ cannot be the maximum. Therefore, the maximum is either w_l or $\beta w_h + (1-\beta)\alpha w_h$.

When $w_l/w_h \geq \alpha + \beta - \alpha\beta$, the winning bid $= w_l$, $Price = w_l$, and $CSn = \alpha\beta(v_h - w_l) + \alpha(1-\beta)(v_l - w_l) + (1-\alpha)\beta(w_h - w_l) > 0$.

Therefore, when $w_l/w_h \geq \alpha + \beta - \alpha\beta$, $CSn > CSh > CSb$ and $CSn > CSv > CSb$; $I_n = \{i|V_{ij}(n) - P_{ij}(n) > 0\}$, $I_h = \{i|V_{ij}(h) - P_{ij}(h) > 0\}$ and $I_v = \{i|V_{ij}(v) - P_{ij}(v) > 0\}$ with $I_v, I_h \subset I_n$.

b) For $v_l < w_h$, winning bid $= max\{w_l, \alpha\beta v_h, \alpha v_l + \beta(1-\alpha)v_l, \beta w_h$. From the initial conditions, we know that $\beta v_h \leq v_l$ implies that $w_l \geq \beta w_h$, therefore βw_h cannot be the maximum. Also, $\alpha v_h \leq w_h$ implies that $\beta w_h \geq \alpha\beta v_h$, further implying that $w_l \geq \alpha\beta v_h$; it follows that $\alpha\beta v_h$ cannot be the maximum. Therefore, the maximum is either w_l or $\alpha v_l + \beta(1-\alpha)v_l$.

When $w_l/v_l \geq \alpha + \beta - \alpha\beta$, the winning bid is $= w_l$, $Price = w_l$, and $CSn = \alpha\beta(v_h - w_l) + \alpha(1-\beta)(v_l - w_l) + (1-\alpha)\beta(w_h - w_l) > 0$.

Therefore, when $w_l/v_l \geq \alpha + \beta - \alpha\beta$, $CSn > CSh > CSb$ and $CSn > CSv > CSb$; $I_n = \{i|V_{ij}(n) - P_{ij}(n) > 0\}$, $I_h = \{i|V_{ij}(h) - P_{ij}(h) > 0\}$ and $I_v = \{i|V_{ij}(v) - P_{ij}(v) > 0\}$ with $I_v, I_h \subset I_n$.

4) When $\beta v_h \geq v_l$, $\alpha v_h \geq w_h$ and $w_h/v_h \leq \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$ or $v_l/w_h \leq \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$, consumers are indifferent.

The region is characterized by the following conditions: $\beta v_h \geq v_l$ and $\alpha v_h \geq w_h$. v_l can be lower or greater than w_h.This implies the following outcomes in the different scenarios:

- Horizonta Information: Winning Bid $= \beta v_h$, $Price = v_h$, $CSh = \beta(v_h - v_h) = 0$.

- Vertical Information: Winning Bid $= \alpha v_h, \alpha v_l$, $Price = v_h, v_l$, $CSv = \alpha\beta(v_h - v_h) +$

$\alpha(1-\beta)(v_l - v_l) = 0.$

- No Information: we have two different maximization problems depending on whether $v_l \geq w_h$ or $v_l < w_h$.

  a) For $v_l \geq w_h$, winning bid $= max\{w_l, \alpha\beta v_h, \alpha v_l, \beta w_h + (1-\beta)\alpha w_h\}$. From the initial conditions, we know that $\alpha v_h \geq w_h$ implies that $w_l \leq \alpha v_l$, therefore w_l cannot be the maximum. Also, $\beta v_h \geq v_l$ implies that $\alpha v_l \leq \alpha\beta v_h$; as a consequence, αv_l cannot be the maximum. Therefore, the maximum is either $\alpha\beta v_h$ or $\beta w_h + (1-\beta)\alpha w_h$.

  When $w_h/v_h \leq \frac{\alpha}{\alpha+\beta-\alpha\beta}$, the winning bid $= \alpha\beta v_h$, $Price = v_h$, and $CSn = \alpha\beta(v_h - v_h) = 0$.

  Therefore, when $w_h/v_h \leq \frac{\alpha}{\alpha+\beta-\alpha\beta}$: $CSh = CSv = CSn = CSb = 0$; $I_n = \{i|V_{ij}(n) - P_{ij}(n) > 0\}$, $I_h = \{i|V_{ij}(h) - P_{ij}(h) > 0\}$ and $I_v = \{i|V_{ij}(v) - P_{ij}(v) > 0\}$ with $I_v = I_h = I_n = 0$.

  b) For $v_l < w_h$, winning bid $= max\{w_l, \alpha\beta v_h, \alpha v_l + \beta(1-\alpha)v_l, \beta w_h\}$.From the initial conditions, we know that $\alpha v_h \geq w_h$ implies that $\beta w_h \leq \alpha\beta v_h$, therefore w_h cannot be the maximum. Also, $\beta v_h \geq v_l$ implies that $w_l \leq \beta w_h$; as a consequence, αv_l cannot be the maximum. Therefore, the maximum is either $\alpha\beta v_h$ or $\alpha v_l + \beta(1-\alpha)v_l$.

  When $v_l/v_h \leq \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$, the winning bid $= \alpha\beta v_h$, $Price = v_h$, and $CSn = \alpha\beta(v_h - v_h) = 0$.

  Therefore, when $v_l/v_h \leq \frac{\alpha\beta}{\alpha+\beta-\alpha\beta}$: $CSh = CSv = CSn = CSb = 0$; $I_n = \{i|V_{ij}(n) - P_{ij}(n) > 0\}$, $I_h = \{i|V_{ij}(h) - P_{ij}(h) > 0\}$ and $I_v = \{i|V_{ij}(v) - P_{ij}(v) > 0\}$ with $I_v = I_h = I_n = 0$.

References

[1] Acquisti, A., (2004). Privacy and security of personal information: Economic incentives and technological solutions. In J. Camp and S. Lewis, editors, The Economics of Information Security.

[2] Acquisti, A., Gross, R., (2009). Predicting social security numbers from public data. In Proceedings of the National Academy of Science

[3] Acquisti, A., Varian, H., (2005). Conditioning Prices on Purchase History. Marketing Science, 24 (3): 1-15.

[4] Anderson, S.P., Coate, S. (2005). Market Provision of Broadcasting: A Welfare Analysis. Review of Economic Studies, 72(4):947-972.

[5] Anderson, S.P., De Palma, A. (2009). Information Congestion. RAND Journal of Economics, 40:688-709.

[6] Armstrong, M. (2006). Competition in Two-Sided Markets. RAND Journal of Economics, 37: 669-691

[7] Athey, S., Ellison, G. (2011). Position Auctions with Consumer Search. Quarterly Journal of Economics, 126:1213-1270.

[8] Athey, S., Guns, J., (2010). The Impact of Targeting Technology on Advertising Markets and Media Competition. American Economic Review, 100(2):608-613.

[9] Bagwell, K., (2001). The Economic Analysis of Advertising, mimeo.

[10] Bergermann, D., Bonatti, A. (2011). Targeting in Advertising Markets: Implications for Offline versus Online Media. RAND Journal of Economics, 42:417-443.

[11] Butters, G.R. (1977). Equilibrium Distribution of Sales and Advertising Prices. Review of Economic Studies, 44:465-491.

[12] Chen, Y., Berkhin, P., Anderson, B., Devanur, N. R. (2011). "Real-time bidding algorithms for performance-based display ad allocation". In Proceedings of the 17th ACM SIGKDD international conference on Knowledge discovery and data mining (pp. 1307-1315). ACM.

[13] Comanor, W.S., Wilson, T.A., (1979). The Effect of Advertising on Competition: a Survey. Journal of Economic Literature, 17 (2): 453-476.

[14] De Corniere, A., De Nijs, R. (2013). Online Advertising and Privacy, mimeo.

[15] De Corniere, A. (2013). Search Advertising. WP

[16] Edelman, B., Ostrovsky, M., Schwarz, M. (2005). "Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords" (No. w11765). National Bureau of Economic Research.

[17] Eliaz, K., Spiegler, R. (2011). A Simple Model of Search Engine Pricing. Economic Journal, 121.

[18] Esteban L., Gil A, Hernandez, J.M., (2001). Informative Advertising and Optimal Targeting in a Monopoly. The Journal Of Industrial Economics, 49 (2): 161-180.

[19] Esteban L., Hernandez, J.M. (2007). Strategic Targeted Advertising and Market Fragmentation. Economics Bulletin,Vol. 12 (10): 1-12.

[20] Goldfarb, A., Tucker, C., (2011). Implications of Online Display Advertising: Targeting and Obtrusiveness. Marketing Science, 1100.0634.

[21] Goldfarb,A., (2014). "What is Different About Online Advertising?". Review of Industrial Organization, 44(20:115-129.

[22] Hagiu, A., Jullien, B. (2011). Why Do Intermediaries Divert Search?. RAND Journal of Economics, 42(2):337-362.

[23] Hann, I-H., Hiu, KL, Lee, SY., Png, I. (2008). Consumer Privacy and Marketing Avoidance: A Static Model. Management Science, 54(6): 1094-1103

[24] Hernandez J.M., (1997). Informative advertising, imperfect targeting and welfare. Economics Letters, 55: 131-137.

[25] Hochman, O., Lusky, I., (1988). Advertising and Economic Welfare: Comment. American Economic Review, 78 (1): 290-6.

[26] Iyer, G., Soberman, D., Villas-boas, J.M., (2005). The Targeting of Advertising. Marketing Science, 24 (3): 461-476

[27] Johnson, J. (2013). Targeted Advertising and advertising avoidance. RAND Journal of Economics, 44(1): 128-144.

[28] Taylor, G. (2013). Search Quality and Revenue Cannibalisation by Competing Search Engines. Journal of Economics and Management Strategy, 22(3), 445-467.

[29] Van Zandt, T. (2004). Information Overload in a Network of Targeted Communication. RAND Journal of Economics, 35:542-560.

[30] White, A. (2013). Search Engines: Left Side Quality versus Right Side Profits. International Journal of Industrial Organization

[31] Yuan, S., Wang, J., Zhao, X. (2013). "Real-time bidding for online advertising: measurement and analysis". In Proceedings of the Seventh International Workshop on Data Mining for Online Advertising (p. 3). ACM.

[32] Zhang, K., Katona, Z., (2012). Contextual Advertising. Marketing Science, 31 (6): 980-994.